A Path Integration Approach to Relativistic Finite Density Problems and Its Particle Content

S. Ying

Physics Department, Fudan University
Shanghai 200433, China
(February 1, 2008)

A path integration formulation for the finite density and temperature problems is shown to be consistent with the thermodynamics using an 8 component "real" representation for the fermion fields by applying it to a free fermion system. A relativistic quantum field theory is shown to be smoothly approached at zero temperature by a real-time thermal field theory so derived even at a finite density. The analysis leads to a new representation for the fermion fields which is shown to be inequivalent to the conventional 4 component theory at the quantum level by having a mirror universe with observable effects and to be better behaved at short distances.

The relativistic quantum field theory (QFT) at finite density and temperature is currently a viable tool to study the real-time processes of a relativistic system under extremely conditions like in heavy ion collisions, in astronomical and cosmological processes, etc. A sufficiently general formulation of the relativistic QFT at conditions different from zero temperature and density is necessary so that it can handle the cases of non-perturbative spontaneous symmetry breaking and particle production and is consistent with thermodynamics in real-time. Such an extension is not trivial.

One of the non-perturbative treatments of the relativistic QFT is based upon Feynman–Matthews–Salam (FMS) path integration [1]. The FMS formalism expresses time evolution between initial and final states in terms of an integration over paths that connect the initial and final states with a weight determined by the action of each particular path. The results of the path integration at a formal symbolic level are not uniquely defined in the Minkowski space-time due to the presence of singularities. Their uniqueness is determined by the fact that the derivation of the FMS path integration representation of the evolution operator implies a particular ordering of the intermediate states which defines how the singularities of the formal results are going to be handled. Thus the FMS path integration formalism is defined by not only the formal expressions that contain singularities but also by the "causal structure" following the time ordering of the physical intermediate states.

One form of the thermal field theory (TFT) was developed in Ref. [2] in the Euclidean space-time with the time variable playing the role of the inverse temperature. Such a formalism, which is based on the periodic (antiperiodic) boundary condition for bosons (fermions) fields, provides the correct thermodynamics for a noninteracting system. The periodic (antiperiodic) boundary condition for the field operators is explicitly written as

$$\widehat{\psi}(t - i\beta) = \eta e^{-\beta\mu}\widehat{\psi}(t), \tag{1}$$

where β is the inverse temperature, $\eta = \pm 1$ for bosons and fermions respectively and μ is the chemical potential. The physical responses of the system to external stimulations that happen in real time can be obtained by a proper analytical continuation. Another formulation of the same problem based upon Eq. 1 can be derived by a distortion of the Matsubara time contour, which goes straightly from 0 to $-i\beta$, to a contour that contains the real time axis extending from negative infinity to positive infinity and returning contour below the real time axis somewhere between 0 and $-i\beta$ that parallels the real time axis (see, e.g., Ref. [3]).

Consistency requires that the results of the TFT go smoothly to that of a relativistic QFT in real time at zero temperature (or $\beta \to \infty$). The question is whether or not it actually happens, especially for fermions. To investigate such a question, let us study the thermodynamics of a free fermion system at finite density and zero temperature using relativistic QFT defined by a FMS path integration formalism.

The thermodynamics of a system with variable fermion number density is determined by the grand potential Ω defined in the following

$$e^{-\beta\Omega} = < e^{-\beta(\widehat{H}_0 - \mu\widehat{N})} >, \tag{2}$$

where $\widehat{N}$ is the fermion number density operator and $\widehat{H}_0$ is the (free) Hamiltonian of the system. The result for Ω is well known from elementary statistical mechanics; it is $U - TS - \mu N$ with U the internal energy, T the temperature, S the entropy and N the average particle number of the system.

In the quantum field theoretical investigation of the same system at zero temperature, there is another representation for the right hand side (r.h.s.) of Eq. 2 in terms of Feynman–Matthews–Salam [1] path integration

$$\lim_{\beta\to\infty} < e^{-\beta\widehat{K}} >= const \times \int [D\psi][D\bar\psi]e^{-S_E}, \tag{3}$$

where $\widehat{K} \equiv \widehat{H}_0 - \mu\widehat{N}$, "*const*" is so chosen that $\Omega = 0$ at zero fermion density. S_E is the Euclidean action of the system; extending the rules of Ref. [4] to finite density cases, it can be obtained from the Minkowski action for the system by a continuous change of the metric. The result is

$$S_E = \int d^4x\bar\psi \left(i\slashed\partial + \mu\gamma^5 - m \right)\psi, \tag{4}$$

where x^μ is the (4-dim) Euclidean space-time coordinates, $\gamma_E^0 = i\gamma^5$ and m is the mass of the fermion. Ω is obtained by identifying β with x_4 (the Euclidean time) that confines the system in the temporal direction before the thermodynamic limit. From Eq. 4, the path integration can be carried out immediately. The result [5] is

$$\Omega' = -\lim_{\beta\to\infty}\frac{1}{\beta}\text{LnDet}\gamma_E^0 \left(i\slashed\partial + \mu\gamma^5 - m \right) - \ln(const)$$

$$= -\lim_{V\to\infty} 2V \int_0^\infty \frac{d\boldsymbol{p}}{2\pi^2}\boldsymbol{p}^2 \int_{-\infty}^{+\infty} \frac{dp_4}{2\pi} \ln\frac{p_+^2 + m^2}{p^2 + m^2} \tag{5}$$

with the Euclidean 4 momenta $p^\mu = (p_4, \boldsymbol{p})$ and $p_+^\mu = (p_4 - i\mu, \boldsymbol{p})$. The result for Ω' is finite despite the infinite integration in momentum. For example, in case of zero mass $PV = -\Omega' = V\mu^4/12\pi^2$, which is expected from the elementary statistical mechanics.

The Euclidean energy p_4 can be treated as a complex number so that the p_4 integration in the complex Minkowski energy p_0 plane is represented by contour C_E in Fig. 1. In the study of the real-time processes of the system, Minkowski action should be used. The original p_0 integration contour C_{FMS} corresponding to the FMS path integration in the Minkowski space-time is also shown in Fig. 1. There are also a pair of contours C_+ and C_- shown in Fig. 2, called the quasiparticle contour, that contribute to the real-time response of the system in the $\beta \to \infty$ limit. Contours C_{FMS}, C_E, C_+ and C_- in Fig. 2 belong to the same topological class of contours having the same FMS causal structure. Consistency requires the equivalence of the set of contours C_{FMS}, C_E, C_+ and C_- for the physical quantities. Eq. 5 unfortunately fails to meet this requirement due to the fact that the imaginary part of the logarithmic function falls off as $O(\mu/|p_0|)$ on the physical p_0 sheet (the imaginary part of the logarithmic function on the edges of its branch cuts on the physical sheet is shown in Fig. 1). This causes the results obtained from doing the p_0 integration on the above mentioned set of contours different from each other since the integration on the sections of the large circle of contours that connecting them has a non-vanishing value.

Explicit computation of Eq. 5 on contour C_+ where only the imaginary part of the integrand with value $\pm i\pi$ contributes (see Fig. 1), reduces to a form differs quite drastically from the expected grand potential of thermodynamics, which is $\Omega = U - \mu N$. In fact, it diverges.

The cause of the above mentioned problems is found to be related to the asymmetric nature of the 4 component representation of the fermion fields with respect to particles and antiparticles. An 8 component "real" representation for the fermion fields can be used [6]. For a spin $1/2$ particle with one flavor, its 8 component spinor Ψ can be written as $\Psi = (\psi_1^T, \psi_2^T)^T$ with the reality condition given by

$$\Psi = \mathcal{M}\bar\Psi^T, \tag{6}$$

where superscript "T" represents the transpose. Here $\mathcal{M} = O_1 \otimes C$ with C the ordinary charge conjugation operator and O_1 the first of three Pauli matrices $O_{1,2,3}$ acting on the upper (ψ_1) and lower (ψ_2) 4 components of Ψ. In the Minkowski space-time, Ψ transforms in the same way as ψ_1 or ψ_2 under the Poincaré group, which commutes with O_i ($i = 1, 2, 3$). Covariant operators can thus be constructed in the same way as in the 4 dimensional representation. For example, the bilinear terms in Ψ can be written as $\bar\Psi\Gamma\otimes O_\lambda\Psi$, where Γ is any covariant matrix in the 4 dimensional Dirac space, $\lambda = \{0, 1, 2, 3\}$ and $O_0 = 1$. Only the matrices satisfying the antisymmetric condition $(\mathcal{M}\Gamma\otimes O_\lambda)^T = -\mathcal{M}\Gamma\otimes O_\lambda$ can be selected since Ψ is a collection of anticommuting Grassmanian numbers in the path integration language.

In terms of Ψ, the invariant action S_E (Eq. 4) for a free massive fermion system in the Euclidean space is $S_E = \frac{1}{2}\int d^4x\bar\Psi \left(i\slashed\partial + \mu\gamma^5 O_3 - m \right)\Psi$ and Eq. 3 becomes $\lim_{\beta\to\infty} < e^{-\beta\widehat{K}} >= const \times \int[D\Psi]e^{-S_E}$ where the $\bar\Psi$ degrees of freedom are not functionally integrated due to the reality condition Eq. 6. The path integration can again be easily evaluated:

$$\Omega = -\lim_{V\to\infty} V \int \frac{d^4p}{(2\pi)^4} \ln\frac{(p_+^2 + m^2)(p_-^2 + m^2)}{(p^2 + m^2)^2} \tag{7}$$

with the same order of integration as Eq. 5. Here $p_-^\mu = (p_4 + i\mu, \boldsymbol{p})$. Eq. 7 and Eq. 5 have an identical value on the contour C_E. They differ on contours $C_\pm$ because the large p_0 behavior of the imaginary part of the logarithmic function in Eq. 7 is of order $O(\mu^2/|p_0|^2)$ on the physical sheet, which allows the equivalence between the set of contours C_{FMS}, C_E, C_+ and C_-. By following the C_+ contour shown in Fig. 1, it is simple to show that the resulting r.h.s. of Eq. 7 is finite and unique, namely, $U - \mu N$. It is the zero temperature grand potential for a free fermion system at density $\bar{n} = N/V$ expected from thermodynamics.

The correct grand potential for a free system in real-time can be obtained using a 4 component representation for the fermion fields in the TFT. The standard method do a Matsubara summation over discrete (imaginary) energies implied by Eq. 1 for the 4 component Dirac spinor ψ. Such a boundary condition for ψ results, however, in different analytic structure for the path integration than that for the FMS path integration in the real time. The grand potential in the Matsubara formalism is

$$\Omega = - \lim_{V \to \infty} \frac{2V}{\beta} \int \frac{d^3 p}{(2\pi)^3}$$
$$\sum_{n=-\infty}^{\infty} \ln \frac{m^2 + \boldsymbol{p}^2 - \left[i(2n+1)\pi\beta^{-1} + \mu\right]^2}{m^2 + \boldsymbol{p}^2 - \left[i(2n+1)\pi\beta^{-1}\right]^2}. \tag{8}$$

The sum over Matsubara frequencies can be evaluated by using a contour integration. For this case, it is $2\pi i\beta^{-1} \sum_n f(z_n) = \int_{C_0} dz \tanh[\beta z/2] f(z)$, where $f[z_n \equiv i(2n+1)\pi\beta^{-1}]$ is the logarithmic function in Eq. 8, C_0 is shown in Fig. 2 and the proper $f(z)$ that gives the correct thermodynamics is $f(z) = \ln[m^2 + \boldsymbol{p}^2 - (z + \mu)^2] - \ln[m^2 + \boldsymbol{p}^2 - z^2]$. Since $f(z)$ approaches to zero fast enough in the $|z| \to \infty$ limit and is analytic in the complex z plane excluding the real axis, the integration over z along contour C_0 is equivalent to the sum of integration over contours C_+ and the negative of C_-, which gives $\Omega = U - TS - \mu N = -PV$ expected from thermodynamics.

The reason why the FMS path integration evaluation for the r.h.s. of Eq. 2 using a 4 component fermion fields fails to give the correct thermodynamics at zero temperature can be understood by a comparison of Figs. 1 and 2. The causal structure of the FMS path integration restricts the p_0 integration for the partition function to be within the class of contours shown in Fig. 1. It lacks the contour pair C_+ and C_- implied by the boundary condition Eq. 1 shown in Fig. 2, which are necessary components to given the correct thermodynamics. With the 8 component "real" spinor Ψ to represent the fermion fields, the effects of the C_- are provided by the lower 4 component ψ_2 of Ψ.

We are ready to develop a real-time field theory at finite density and temperature using the 8 component "real" representation for the fermion fields. The boundary condition Eq. 1 is written in an equivalent form. For the fermions interested in this study, it can be expressed as $\widehat{\Psi}(t) = -e^{\beta \widehat{K}} \widehat{\Psi}(t) e^{-\beta \widehat{K}}$. It is equivalent to Eq. 1 due to the fact that the conserved $\widehat{N}$ commutes with the total Hamiltonian $\widehat{H}$, which allows the factorization of the action of $\beta\mu\widehat{N}$ and $\beta\widehat{H}$ in the exponential of $\beta\widehat{K}$ with the $\exp(\beta\mu)$ factor[1] in Eq. 1 the result of the action of $\exp(\beta\mu\widehat{N})$ and $\exp(-\beta\mu\widehat{N})$ on ψ from left and right. To handle non-perturbative symmetry breaking cases, the commutativity of $\widehat{H}$ and $\widehat{N}$ shall not be imposed at this level of development but rather at later stages as dynamical constraints.

The real-time thermal field dynamics in the 8 component "real" representation for fermion fields can be constructed along the same line as that of Refs. [3], [5], [7]– [9]. Compared to those developments, the only formal differences of the present approach are 1) the Kubo–Martin–Schwinger (KMS) boundary condition for the contour propagator do not contain the $\exp(\beta\mu)$ factor in the present approach, 2) the effects of μ is hidden in the energy variable within the propagator (namely, the time evolution is generated by $\widehat{K}$ not $\widehat{H}$) here and 3) the symmetry factor for a Feynman diagram in a perturbation expansion is different from the 4 component representation for the fermion fields due to Eq. 6, which also allows the Wick contraction between two Ψs [6]. The matrix form of contour propagator[2] for a fermion is

$$\mathcal{S}(p) = M \begin{pmatrix} S^+ & 0 \\ 0 & S^- \end{pmatrix} M, \tag{9}$$

where the retarded and advanced propagators S^+ and S^- are $S^\pm(p) = i/(\not{p} + \gamma^0\mu O_3 - m \pm i\gamma^0\epsilon)$ and $M = \text{sgn}(p_0)\sqrt{1 - n(p_0)} - i\sigma_2\sqrt{n(p_0)}$ with $n(e) = 1/[exp(\beta e) + 1]$ and σ_2 the second Pauli matrix. The poles of the above propagator at zero temperature are located above the contour C_{FMS} if they are negative and below it otherwise. It can be compared to the conventional real-time approach with the correct causal structure [10,11] at zero

---

[1]It should be $\exp(\beta\mu O_3)$ in the new representation.
[2]The returning time contour parallels the real time axis is chosen to be located half way between 0 and $-i\beta$.

temperature, in which the poles of the fermion propagator are located above the p_0 integration contour if they are on the left of μ or else below it. The real-time partition function obtained here for the system is identical to the Matsubara method since it has a better large p_0 behavior at finite density.

Let us turn to the study of local bilinear operators constructed from two fermion fields of the form $\widehat{O} = \bar{\psi}(x)\Gamma\psi(x)$ in a finite density situation where Γ is certain matrix acting on ψ. A product of two field operators is in general singular and non-unique. A definition of such a potentially divergent product is $\widehat{O} = \lim_{\delta_\mu \to 0} \bar{\psi}(x + \delta)\Gamma\psi(x)$, where δ_μ is a 4-vector with, e.g., $\delta_0 < 0$. The ground state (vacuum) expectation value of $\widehat{O}$ can thus be computed from the fermion propagator $\mathcal{S}$ by closing the integration contour on real p_0 axis in the lower half of the complex p_0 plane. Since there is no poles and cuts off the real p_0 axis except the ones on the imaginary p_0 axis due to the thermal factor, which should be excluded (see Fig. 2), it can be deformed to C_R to include the poles and cuts of the retarded propagator S^+:

$$< \widehat{O} > \; = -\mathrm{tr} \int \frac{d^3p}{(2\pi)^3} \int_{C_R} \frac{dp_0}{2\pi} \mathcal{S}_F^{11}(p)\Gamma, \tag{10}$$

with "tr" denoting the trace over internal indices of the fermion fields. C_R reduces to C_+ as $\beta \to 0$. For a free system, there are only poles in $\mathcal{S}^{11}$ so that Eq. 10 can be expressed as

$$< \widehat{O} > \; = i \int \frac{d^3p}{(2\pi)^3} \sum_k [1 - n(p_k^0)]\mathrm{tr}\mathrm{Res}S^+(p_k)\Gamma, \tag{11}$$

where the sum is over all poles p_k^0 of $S^+(p)$ and "Res" denotes the corresponding residue. The difference between the 8 component theory here and the 4 component one also manifests here. For example, the conserved fermion number density $\bar{n}$ corresponding to current $j^\mu = \frac{1}{2}\bar{\Psi}\gamma^\mu O_3\Psi$ is $\bar{n} = \frac{1}{\pi^2} \int_0^\infty dp p^2 \left[f_p^{(-)} - f_p^{(+)} \right]$ with $f_p^{(\pm)} = 1/[e^{\beta(E_p \pm \mu)} + 1]$. It is same as the one in elementary statistical mechanics. On the other hand, $\bar{n}$ computed in the conventional approach using such a point split definition of fermion number density is divergent and different from each other for $\delta_0 > 0$ and $\delta_0 < 0$, which means that it is not even consistent with relativity it starts with for a space like δ_μ whose time component can has different sign in different frames. It can be made finite and unique only after an arbitrary subtraction.

The particle content of the 8 component theory can be found by studying pole structure of the fermion propagator together with the consideration of the FMS causal condition. The time dependence of the propagator at zero temperature is $\mathcal{S}(t, \boldsymbol{p}) = \int_{C_{FMS}} \frac{dp_0}{2\pi} e^{-ip_0t}\mathcal{S}(p_0, \boldsymbol{p})$, which, in case of $t > 0$, can be evaluated on the contour C_+ with a result

$$\mathcal{S}(t, \boldsymbol{p}) = \theta(E_p - \mu)\Lambda_p^{1+}e^{-i(E_p - \mu)t} + \theta(\mu - E_p)\Lambda_p^{2-}e^{-i(\mu - E_p)t} + \Lambda_p^{2+}e^{-i(E_p + \mu)t} \tag{12}$$

and, if $t < 0$, can be evaluated on the contour C_- to obtain

$$\mathcal{S}(t, \boldsymbol{p}) = \Lambda_p^{1-}e^{i(E_p + \mu)t} + \theta(\mu - E_p)\Lambda_p^{1+}e^{i(\mu - E_p)t} + \theta(E_p - \mu)\Lambda_p^{2-}e^{i(E_p - \mu)t}. \tag{13}$$

Here $\Lambda_p^{r\pm} = P_r(\pm\gamma^0 E_p - \boldsymbol{\gamma} \cdot \boldsymbol{p} + m)/2E_p$, $P_1 = (1 + O_3)/2$ and $P_2 = (1 - O_3)/2$ are projection operators.

The FMS causal structure (or boundary condition) in the present theory can be simply putted as: 1) excitations with $p_0 > 0$ that propagate *forward in time* correspond to particles or antiholes and 2) those with $p_0 < 0$ that propagate *backward in time* correspond to antiparticles or holes. The quantization of the 8 component fermionic field then follows naturally. In case of zero density ($\mu = 0$), $\widehat{\Psi}(x)$ can be written as

$$\widehat{\Psi}(x) = \sum_{rps} \frac{1}{2E_p} \left[U_{rps}e^{-ip \cdot x}\hat{b}_{rps} + V_{rps}e^{ip \cdot x}\hat{d}^\dagger_{rps} \right]. \tag{14}$$

Here s is the spin index, $r = 1, 2$, U_{rps} and V_{rps} are 8 component spinors that satisfy $(\not{p} - m)U_{rps} = 0$, $O_3U_{rps} = (3 - 2r)U_{rps}$, $(\not{p} + m)V_{rps} = 0$, $O_3V_{rps} = (3 - 2r)V_{rps}$ and the non vanishing anticommutators between $\hat{b}_{rps}$ and $\hat{d}_{rps}$ are $\left\{ \hat{b}_{rps}, \hat{b}^\dagger_{r'p's'} \right\} = 2E_p\delta_{pp'}\delta_{ss'}\delta_{rr'}$, $\left\{ \hat{d}_{rps}, \hat{d}^\dagger_{r'p's'} \right\} = 2E_p\delta_{pp'}\delta_{ss'}\delta_{rr'}$, which lead to a canonical quantization of $\widehat{\Psi}$. The particle states are obtained from the vacuum state $|0\rangle$, which is annihilated by the $\hat{b}$s and $\hat{d}$s above, by the action of $\widehat{\Psi}$ and $\widehat{\bar{\Psi}}$ on it. Albeit proper counting has already been given to the intermediate states during its evolution [6], these particle states are not all detectable since the reality condition Eq. 6 has to be imposed on the detector. The operator reality condition for the fermion field $\widehat{\Psi}$ are given by the mirror reflection operator $\widehat{\mathcal{M}}$, which is defined by $\widehat{\mathcal{M}}\widehat{\Psi}^T\widehat{\mathcal{M}} \equiv \mathcal{M}\widehat{\Phi}^T$ with $\widehat{M}^2 = \widehat{I}$, where $\widehat{\bar{\Phi}}$ is obtained from $\widehat{\Psi}$ by the following replacement: $\widehat{\mathcal{M}}\hat{b}_{rps}\widehat{\mathcal{M}} = \hat{d}_{rps}$. The

operator reality condition corresponding to Eq. 6 is then $\widehat{\Psi} = \mathcal{M}\widehat{\overline{\Phi}^T}$. The vacuum state satisfies $\widehat{P}_{phys}|0\rangle = |0\rangle$. So the physical states satisfying the operator reality condition are those ones symmetric in $r = 1$ and $r = 2$ projected out by $\widehat{P}_{phys} = (\widehat{I} + \widehat{\mathcal{M}})/2$. In finite density case, the annihilation and creation operators for the holes (in the Fermi sea) has to be introduced, which is straightforward.

Compared to the 4 component theory, which has only one fermionic excitation excitation mode with an effective energy $\epsilon_- = E_p - \mu$ ($\mu > 0$) for particle, which is the one with $r = 1$ in Eq. 12, Eq. 12 has a new fermionic excitation mode with $r = 2$ and an effective energy $\epsilon_+ = E_p + \mu$ possessing a common 3 momentum range, spin structure and opposite charge against the $r = 1$ one. This excitation mode is much larger in energy than the $r = 1$ one (ϵ_-) in a non-relativistic system like the electron gas, it can be comparable in relativistic systems like the quark gluon plasma where the bare mass of the fermion is very small relative to its momentum. In addition, there is another particle mode with $r = 2$ and effective energy $\mu - E_p$. Its effects will be further discussed in the following. First, let us mention that the $r = 1$ and $r = 2$ parts in a free theory decouple and give, separately, identical results for observables due to the charge conjugation symmetry. They belong to two non communicating opposite charged mirror universes. Therefore the differences between the 8 component theory and the 4 component one can not show themselves if the $r = 1$ and $r = 2$ parts are not produced in coherent states of the form

$$|\phi\rangle = \cos\phi|1\rangle + e^{i\delta}\sin\phi|2\rangle \tag{15}$$

with $\phi \neq 0$ and δ arbitrary, which does not separat apart during its evolution in time. The following is a scenario in which quantum interference effects of the 8 component theory that are not present in the 4 component one can be observed for interacting theories.

The possibility of the existence of vacuum phases in a relativistic massless fermion system induced by a condensation of fermion pairs and antifermion pairs and their possible relevance to physical hadronic system are studied in Refs. [6,12]. Suppose that the β and ω phases discussed in Refs. [6,12] that mix the $r = 1$ and $r = 2$ mirror universes are produced in, e.g. a heavy ion collision, or an astronomical object, then a quantum state of the form given by Eq. 15 with a fixed δ and $\phi \neq 0$ can be produced in the collision region [6]. Such a state contains $r = 1$ and $r = 2$ parts which have the same 3 momentum $\boldsymbol{p}$. After this state leaves the collision region where the density is effectively zero to propagate to the detector, they remains in a mixed state of the form given by Eq. 15 with $r = 1$ and $r = 2$ parts having the same 3 momentum, which guarantees that these two parts can propagate together without been spatially separated on their way to the detector. To distinguish these two components of the particle in the detector, one can generate a non zero electric potential A^0 by, say, putting the detector into a hollow charged conductor. Inside this $A^0 \neq 0$ region, the $r = 1$ and $r = 2$ pieces begin to have a different time dependence $\exp[-i(E_p \mp qA^0)t]$ with q the charge of the particle, which can generates detectable beats in the counting rate. To show this more explicitly, let us consider the counting rate of finding a particle in a space volume $\Delta\Omega$; $P_{\Delta\Omega} = \int_{\Delta\Omega} d^3x\,|\langle x\,|\,\boldsymbol{p}, \phi\rangle|^2$ is

$$\int_{\Delta\Omega} d^3x \left[\cos^2\phi\,|\langle x\,|\,\boldsymbol{p}, 1\rangle|^2 + \sin^2\phi\,|\langle x\,|\,\boldsymbol{p}, 2\rangle|^2 \right.$$
$$\left. + \sin\phi\cos\phi\left(\langle x\,|\,\boldsymbol{p}, 1\rangle\langle x\,|\,\boldsymbol{p}, 2\rangle^* e^{-i\delta} + c.c\right)\right], \tag{16}$$

where $\boldsymbol{p}$ is the 3 momentum of the excitation, c.c. denotes complex conjugation and the single particle physical state at x satisfies $\langle x\,|\widehat{P}_{phys} = \langle x\,|$. It is ready to see that the third term depends on time with an universal beat frequency $2qA^0$ for all directly pruduced particles with the same q when $\phi \neq 0$. This effect is absent if the conventional 4 component theory is used.

The other excitation mode with $r = 2$ in Eq. 12 that corresponds to particle has a different spin structure and momentum range from the $r = 1$, ϵ_- excitation mode. This excitation mode is not expected to has any interference effects with the $r = 1$, ϵ_- excitation mode. Their effects on physical observables are addative on the probability level instead of the amplitude level. So they simply reproduce the effects of those in the 4 component theory due to charge conjugation symmetry of the system.

In summary, we developed an 8 component theory for the relativistic finite density systems that has better short distance behavior than the corresponding 4 component one. The introduction of a mirror universe is found to be necessary. The link between the possible mirror universe and ours can be established and remains to be searched for in the real world phenomena of heavy ion collisions, mechanism for the CP violation, baryogenesis in the early Universe, the origin of dark matter, etc.

[1] R. P. Feynman and A. R. Hibbs, *Quantum Mechanics and Path Integrals* (McGraw–Hill, New York, 1965); P. T. Matthews and A. Salam, Nuovo Cim. **2**, 120 (1955).

[2] T. Matsubara, Prog. Theor. Phys. **14**, 3551 (1955).

[3] N. P. Landsman and Ch. G. van Weert, Phys. Rep., **145**, 141 (1987).

[4] M. R. Mehta, Phys. Rev. Lett., **65**, 1983 (1990).

[5] J. Kapusta, *Finite–Temperature Field Theory* (Cambridge University Press, 1989).

[6] S. Ying, Phys. Lett.,**B283**, 341 (1992); S. Ying, Ann. Phys. (N.Y.), **250**, 69 (1996); S. Ying, in preparation.

[7] G. W. Semenoff, H. Umezawa, Nucl. Phys. **B220**, 196 (1983).

[8] A. J. Niemi, G. W. Semenoff, Nucl. Phys. **B230**, 181 (1984); Ann. Phys. (N.Y.) **152**, 105 (1984).

[9] R. L. Kobes, G. W. Semenoff, N. Weiss, Z. Phys. **C29**, 371 (1985).

[10] E. V. Shuyak, Phys. Rep. **61**, 71 (1980).

[11] M. Lutz and T. Kunihiro, Z. Phys. **C49**, 123 (1991).

[12] S. Ying, Preprint, hep-ph/9604255.

FIGURES

FIG. 1.   The set of contours belonging to the same FMS class. Contour C_{FMS} is the original FMS contour in the Minkowski space. Contour C_E is the Euclidean contour. Contour C_+ is the quasiparticle contour. $\pm i\pi$ denote the imaginary part of the integrand along the edges of its cuts (thick lines) on the physical p_0 plane.

FIG. 2.   The set of contours that give the thermodynamics. Contour C_0 encloses the Matsubara poles of the integrand. Contours C_+ and C_- are the ones needed in a real time formulation of the thermal field theory in the 4 component representation for fermion fields.
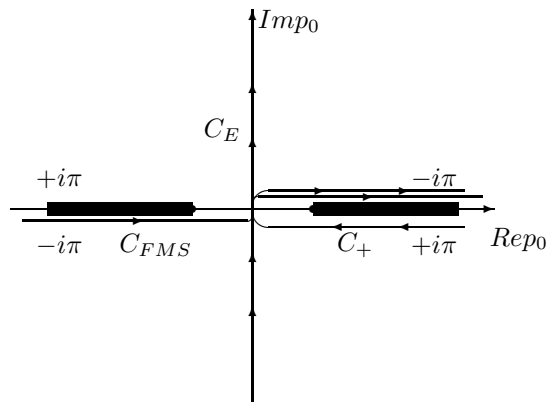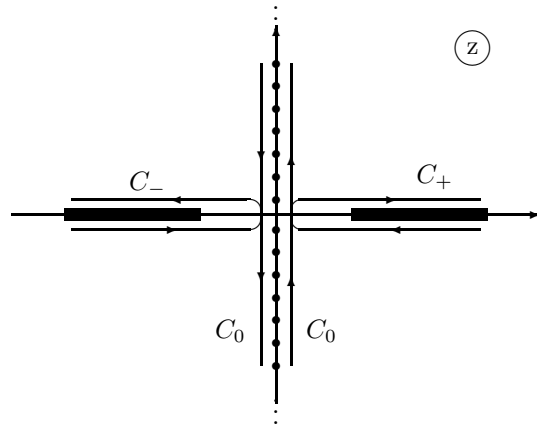
Figure 1

Figure 2